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their parts. $\therefore$ angle BFC=angle at D. $BCF = CBF = DCE$. But BCF, being measured by the arc AF, is double the angle at D, which is measured by half the arc AF; $\therefore \angle BCF = 2\angle BFC$.

The angles BFC, BCF and CBF are together equal to two right angles, $\therefore BCF = \frac{2}{5}$ of two right angles, or $\frac{1}{5}$ of four right angles; $\therefore AF$ is $\frac{1}{5}$ of the circumference. Q. E. D.

SCHOLIA.—1. Taking the angle D as the unit of measurement, we have $D = BFC = FCE = CFE$, $2D = FBC = FCB = BFD = DCE = DEC$; $3D = CEF = BCE = FCD = FBA$.

2. $DE = DC = CF = BF =$ radius; $BC = CE = EF =$ greater segment of radius. Also the triangles CDE, CEF, DCF, BCF and BFD, i. e., all the triangles in the figure, are isocceles.

3. Laying off from F towards D an arc equal to AF, we get a remainder DG which is $\frac{1}{10}$ of the circumference. We know from Euclid that the chord from D to G is equal to BC; hence BC, CE and EF are each equal to the chord of a decagon, and DE, &c., to the chord of a hexagon. Also, $BD = DF =$ chord of $\frac{3}{5}$ of two right angles, or $\frac{6}{5}$ of one right angle, $\therefore FCD = FBA$ &c. = angle between two consecutive sides of a regular pentagon.

4. Since $BC : CD :: CD : BD$, chord of a regular inscribed decagon : chord of hexagon :: chord of hexagon : chord of 108°.

Putting CD=radius $= 1$, chord $36° = \dfrac{1}{\text{chord } 108°}$; $\therefore \sin 18° = \dfrac{1}{\sin 54°}$. Since $BD = CD + BC$, chord $108° = 1 +$ chord $36°$. $\therefore \sin 54° = .5 + \sin 18°$, as may be seen in a table of natural sines.

To find the length of BC, which we will call x, $x : 1 :: 1 : x+1$; $\therefore x^2 + x = 1$, whence $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5} = BC$; $\therefore BD = DF = +\frac{1}{2} + \frac{1}{2}\sqrt{5}$.

$AF^2 = AD^2 - DF^2 = 4 - \frac{1}{4} - \frac{1}{2}\sqrt{5} - \frac{5}{4} = \frac{5}{2} - \frac{1}{2}\sqrt{5} = BC^2 + CD^2$; $\therefore$ the side of a regular inscribed pentagon is the hypothenuse of a right angled triangle, the other sides being the side of a regular inscribed hexagon and that of a regular inscribed decagon. This has been noticed by Young.

A PROBLEM AND ITS SOLUTION.

BY DR. H. EGGERS, MILWAUKEE, WISCONSIN.

Problem. — Given in a plane three fixed right lines, L_1, L_2, L_3, and in each of them a fixed point, respectively, A_1, A_2, A_3: required a right line M, which shall cut off on L_1, L_2, L_3, three equal distances counting from

A_1, A_2, A_3, that is, the line M shall mark on L_1, L_2, L_3, respectively, three points, B_1, B_2, B_3, so that $A_1B_1 = A_2B_2 = A_3B_3$.

Solution. — Taking on one of the given lines, on L_1, for instance, any number of distances, counting from A_1, and on the line L_2 the sam distances, counting from A_2, then the corresponding terminating points on both lines will form two homographic systems of points, the projecting rays of which envelop a parabola touching L_1 and L_2.

Combining in the same manner L_1 and L_3, a second parabola will be generated, touching L_1 and L_3. These two parabolas have L_1 and the line at infinity L_∞ as common tangents. The other pair of common tangents will evidently solve the problem. For in virtue of the first parabola they severally cut off equal distances on L_1 and L_2, and in virtue of the second parabola they cut off equal distances on L_1 and L_3.

The two required common tangents, T_1 and T_2, can be constructed as follows:

Let X be their point of intersection and Y_∞ the point of intersection of L_1 with the line in infinity, L_∞. These two points, X and Y_∞, constitute one of the conics enveloped by the four common tangents, T_1, T_2, L_1, L_∞.

If we take now an arbitrary point E and draw from E the two pairs of tangents on the two parabolas, and pass also from E two lines through the two points X and Y_∞, these six lines form an involution, of which five lines are given; therefore the sixth line, which must pass through X, can be constructed. Taking now another arbitrary point E_1 and performing the analogous construction, we gain a second line E_1X, which must pass through X. Therefore the point of intersection X of the two tangents T_1 and T_2 can be found. All that remains is, to construct through X the two tangents to one of the parabolas. These will touch the other parabola also.

If the direction of counting the distances from A_1, A_2, A_3 is not determined beforehand, there will be four pairs of solutions possible, of which some may be imaginary. For we may on each of the given lines L_1, L_2, L_3 discern two directions of counting, + and —, the combinations of which will furnish four different cases, viz.;

L_1	+	+	+	—
L_2	+	+	—	+
L_3	+	—	+	+

Of the four other combinations of signs yet possible, any one will coincide with one of the four above given; for example, three minus signs give the same solution as three plus signs, because the variable points of division which mark equal distances on L_1, L_2, L_3, pass at the same time to infinity and leap from there simultaneously on the opposite half of these lines.

Each of the four combinations of signs now gives rise to two solutions, which in regard to method do not differ.

If the distances to be cut off, instead of being equal, shall have given ratios

$$m : n : p$$

the method of solution remains the same.

The solution of this problem becomes more simple when two of the given lines are parallel.

SOLUTIONS OF PROBLEMS IN NUMBER FIVE.

SOLUTIONS of problems in No. 5 have been received as follows:

From Prof. L. G. Barbour, 224; Marcus Baker, 225, 230; Prof. P. E. Chase, 230; Newton Fitz, 221, 230; Henry Gunder, 221, 223, 224, 230; Henry Heaton, 221, 223, 224, 225, 226, 227, 228, 229, 230; W. E. Heal, 221, 227; Prof. E. W. Hyde, 225, 230; Prof. Joseph H. Kershner, 221, 222, 223, 224, 225, 226, 228, 229, 230; Chas. H. Kummell, 221, 224, 227, 228, 229, 230; Prof. D. J. Mc Adam, 223, 224, 228; Prof. Orson Pratt, 221; P. Richardson, 221, 225; Prof. J. Scheffer, 221, 224, 225, 226, 228; E. B. Seitz, 221, 223, 224, 225, 227, 228, 230.

221. "Find the four roots of the equation,

$$x^4+px^3+\frac{7p^2}{4^2}x^2+\frac{6p^3}{4^3}x+q=0."$$

SOLUTION BY PROF. ORSON PRATT, SEN., SALT LAKE CITY, UTAH.

Increase the roots by $\frac{1}{4}p$, and the second and fourth terms vanish, and the resulting equation becomes a quadratic; reducing, and diminishing the roots by $\frac{1}{4}p$, we have

$$x=-\frac{p}{4}+\frac{1}{4\sqrt{2}}\sqrt{\left[-p^2+\sqrt{(9p^4-4^5q)}\right]};$$

$$x=-\frac{p}{4}-\frac{1}{4\sqrt{2}}\sqrt{\left[-p^2+\sqrt{(9p^4-4^5q)}\right]};$$

$$x=-\frac{p}{4}+\frac{1}{4\sqrt{2}}\sqrt{\left[-p^2-\sqrt{(9p^4-4^5q)}\right]};$$

$$x=-\frac{p}{4}-\frac{1}{4\sqrt{2}}\sqrt{\left[-p^2-\sqrt{(9p^4-4^5q)}\right]}.$$